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ABSTRACT

Two sequences of activities were developed to provide instruction on the algorithms for addition and subtraction of two-digit numbers. In the integrated sequence (I) the mechanics of "carrying" and "borrowing" were treated as a single process "regrouping." In the sequential treatment (S) the addition algorithm was developed before the subtraction algorithm. Students of two second-grade classes were randomly reassigned to either group S or group I. Profiles were generated by item sampling. Group means were estimated for addition, subtraction and total performance every three days. Also, on the eighteenth day all children were administered a 20-item achievement test. Overall differences in group performance were not significant. Some differences in performance on operations at specific times were significant and favored group S. (A slightly different prepublication draft of this document was announced as ED 062 195.) (Author/MM)

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TECHNICAL REPORT NO. 222

THE RELATIVE EFFECTIVENESS OF TWO DIFFERENT INSTRUCTIONAL SEQUENCES DESIGNED TO TEACH THE ADDITION AND SUBTRACTION ALGORITHMS

REPORT FROM THE PROJECT ON ANALYSIS OF MATHEMATICS INSTRUCTION

WISCONSIN RESEARCH AND DEVELOPMENT

CENTER FOR COGNITIVE LEARNING

RESEARCH
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LEARNING
DEVELOPMENT

Technical Report No. 222

THE RELATIVE EFFECTIVENESS OF TWO DIFFERENT INSTRUCTIONAL SEQUENCES DESIGNED TO TEACH THE ADDITION AND SUBTRACTION ALGORITHMS

by

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Report from the Project on
Analysis of Mathematics Instruction

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Statement of Focus

The Wisconsin Research and Development Center for Cognitive Learning focuses on contributing to a better understanding of cognitive learning by children and youth and to the improvement of related educational practices. The strategy for research and development is comprehensive. It includes basic research to generate new knowledge about the conditions and processes of learning and about the processes of instruction, and the subsequent development of research-based instructional materials, many of which are designed for use by teachers and others for use by students. These materials are tested and refined in school settings. Throughout these operations behavioral scientists, curriculum experts, academic scholars, and school people interact, insuring that the results of Center activities are based soundly on knowledge of subject matter and cognitive learning and that they are applied to the improvement of educational practice.

This Technical Report is from Phase 2 of the Project on Prototypic Instructional Systems in Elementary Mathematics in Program 2. General objectives of the Program are to establish rationale and strategy for developing instructional systems, to identify sequences of concepts and cognitive skills, to develop assessment procedures for those concepts and skills, to identify or develop instructional materials associated with the concepts and cognitive skills, and to generate new knowledge about instructional procedures. Contributing to the Program objectives, the Mathematics Project, Phase 1, is developing and testing a televised course in arithmetic for Grades 1-6 which provides not only a complete program of instruction for the pupils but also inservice training for teachers. Phase 2 has a long-term goal of providing an individually guided instructional program in elementary mathematics. Preliminary activities include identifying instructional objectives, student activities, teacher activities materials, and assessment procedures for integration into a total mathematics curriculum. The third phase focuses on the development of a computer system for managing individually guided instruction in mathematics and on a later extension of the system's applicability.

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Abstract

Two sequences of activities were developed to provide instruction in using the algorithms for addition and subtraction of two-digit numbers. In the integrated sequence (I) the mechanics of "carrying" and "borrowing" were treated as a single process "regrouping." In the sequential treatment (S) the addition algorithm was developed before the subtraction algorithm.

Students of two second-grade classes were randomly reassigned to either group S or group I. Profiles were generated by item sampling. Group means were estimated for addition, subtraction, and total performance every three days. Also, on the 18th day all children were administered a 20-item achievement test.

Overall differences in group performance, though favoring group S, were not statistically significant. Some differences in performance on operations at specific times were significant and favored group S.

Learning profiles of performance across time indicated addition scores were significantly higher than subtraction scores for both groups. The learning profiles also indicated significant linear trends for both operations while no higher order trends were evident.

Other effects independent of treatment were also evident on the posttest. There were significant differences favoring addition over subtraction, and non-regrouping over regrouping. There was also a significant interaction between the operations and the regrouping behavior required, so that "borrowing" was more difficult for subtraction than "carrying" was for addition.

I
Introduction

The purpose of this study was to compare the relative effectiveness of two instructional sequences designed to teach the addition and subtraction algorithms for two-digit whole numbers. One of these sequences is the traditional sequence of addition followed by subtraction; the other sequence is an integrated presentation of the two tasks. Each sequence was embodied in a set of instructional activities that were used with a randomly selected group of second-grade children.

Data were gathered periodically during instruction and near the completion of the set of activities. Item sampling was used to obtain the periodic measures, but the final test was a conventional test of ability to use the algorithms. Comparisons between the groups were made by testing for differences in the estimated group mean achievement profiles and by testing for differences in group mean performance on the conventional test.

Context of the Study

This study is one of several studies on instructional problems being conducted by the staff of the Analysis of Mathematics Instruction (AMI) Project of the Wisconsin Research and Development Center for Cognitive Learning.

Romberg and Harvey (1969) outlined a curriculum development plan in which a hierarchy of behaviors is used to provide focal points for instructional activities. In general the hierarchy is followed from the bottom up, but the behaviors are grouped rather than being attacked one at a time. Activities are then written to enable the children to achieve the entire set. This development plan is being used to construct a new elementary mathematics curriculum entitled Developing Mathematical Processes (DMP).

The content elements to be sequenced are the behaviors, and the embodiments of these behaviors are activities that may encompass several behaviors. The questions of sequencing concern how behaviors are grouped together, and how the groups of behaviors are sequenced. In particular, the problems have nothing to do with the sequencing of frames, as in some programmed instruction approaches, or with the ordering of examples and rules. While behaviors are probably mastered one at a time, this grouping of behaviors allows the behaviors to be designated as preparatory, mastery, or review with respect to a given activity. An activity may be discovery oriented, expository oriented, or any point in between. It may be designated for a single child or for groups of children. Some activities used in this study are appended to this report.

Related Research

Research relating to the use of integrated sequences has been reported by Newton and Hickey (1965), Short and Haughey (1967), and Gray (1970). While all three of these studies provide evidence to support the use of integrated sequences, only the last two involve grade school subjects. Only the last two studies will be reviewed here.

Short and Haughey (1967) compared the results of using sequences generated by means of a multiple-concept sequencing strategy with sequences generated using a single-concept sequencing strategy. These strategies are defined as follows:

> The multiple-concept strategy presents simple descriptions of several related concepts at the beginning of instruction. Increasingly complex material pertaining to all these concepts is then gradually introduced....
> The single-concept strategy presents one concept at a time, proceeding from a

simple description of the single concept to more complex descriptions of the same concept. After the concept has been presented in all its detail and complexity, a second concept is introduced, described in detail, and then a third concept is introduced, etc.

The type of task used by Short and Haughey is identified as a multiple-discrimination task. It is characterized as a task through which "a student learns to make different responses to similar stimuli that previously evoked an undifferentiated response" (Short & Haughey, 1967). The content for their study was taken from both science concepts and language arts concepts; programmed materials were used with fifth-grade subjects.

Short and Haughey found that groups who received the multiple-concept sequence did better in all comparisons. However, only the science materials produced differences that were statistically significant.

Gray (1970) reports the relative effects on acquisition and retention of mathematics and science behaviors by fifth-grade children that resulted from using an integrated and a nonintegrated learning sequence. The content was identified as three quantitative science behaviors. A task analysis identified a total of 25 behavioral objectives involving both mathematics and science behaviors; the two sequences were selected from this hierarchy. One sequence separated all of the mathematics behaviors from the science behaviors and taught the former first. The other sequence was integrated with respect to these two sets of behaviors so as to emphasize the relationships between the two sets of behaviors. Both sequences of behaviors were incorporated in lessons to be taught in 12 class sessions. Measures of achievement were taken the day following completion of the lessons and again nine weeks later. An analysis of variance was used to test for differences in acquisition and retention of the behaviors. With reference to this analysis Gray concluded that "the integrated learning sequence was generally superior to the non-integrated sequence in facilitating acquisition of the mathematical behaviors for the population defined in this study." No differences in the rate of forgetting were observed.

Summary

Prior evidence supports the use of a multiple-task sequencing strategy for constructing instructional sequences for children at the fifth-grade level, but the evidence is not without qualification. While Short and Haughey dealt specifically with multiple-discrimination tasks, it is not clear that Gray's study involved only such tasks. However, Gray did not deal specifically with procedural chains.

To make the difference between multiple discriminations and procedural chains clear, the following definition adapted from Suppes (1969) is proposed. A procedural chain is a finite sequence of instructions that can be mechanically followed to completion. The difference between multiple discrimination and a procedural chain is in the responses required. The response to multiple-discrimination stimuli is to classify or identify the stimuli, or perhaps even to initiate some action as a result of the classification made. The response to the stimuli of a procedural chain is to initiate and carry out a sequence of responses where the result of one response together with the initial stimuli determine what the next response is to be.

Prior available evidence concerning the use of integrated sequences does not relate directly to subjects in the age range of seven to nine years or to tasks of the kind represented by the algorithms in question. It seems clear that the algorithms must certainly be classified as procedural chains as opposed to concepts or multiple discriminations.

II
Method

Instructional Sequence Construction

The construction of an instructional sequence involves analyzing the content into elements (concepts, behaviors, frames, etc.) and determining the order in which the learner is to interact with them. Accordingly, an instructional sequence is defined as the elements of content in the order in which the learner is to interact with them.

One method of selecting elements of content and establishing guidelines for sequencing them that has found acceptance is task analysis (e.g., Gagné, 1968; Gray, 1970; Kersh, 1967). The analysis of the content results in a set of behavioral objectives (behaviors) arranged in a hierarchy indicating judged dependency relationships among the behaviors; i.e., behavior x is dependent on behavior y if y is judged to be a prerequisite behavior of x.

An instructional sequence may be formed from a hierarchy by grouping the behaviors into ordered sets (hereafter called tasks) and then ordering the tasks. When behaviors of one task are prerequisite to behaviors of another task, the first task may be called subordinate to the second and the instructional order is indicated. Presumably if two tasks should appear as subordinate to each other, a regrouping of the behaviors is necessary to eliminate such cases. If two tasks, X and Y, are not in a subordinate relationship, they will be called coordinate tasks, and their order is not indicated by the hierarchy. The immediate possibilities are X then Y, and Y then X; however, it may be reasonable to integrate tasks X and Y to form a new task, Z.

The integration of coordinate tasks is particularly attractive when the behaviors of the tasks are related in ways that may not be indicated by the hierarchy. For example, they may have similar stimuli or similarities in the responses required, or they may have nondependent relationships in the logical structure of the content involved; many of these facts would not be reflected in the behavioral hierarchy.

Heimer (1969) identified four issues relating to the use of learning hierarchies. These are:

1) How is a learning hierarchy constructed and under what conditions is one considered valid?

2) What is the relationship between an hypothesized learning hierarchy and the associated presentation sequence for instruction?

3) What are the "intellectual skills" that make up a learning hierarchy?

4) What is the connection, if any, between the (logical) structure of the content and the design of the associated learning hierarchy?

This study bears particularly on the second issue.

The Instructional Sequences

For this study the coordinate tasks were the two sets of behaviors associated with learning the usual algorithms for addition and subtraction of two two-digit whole numbers. An earlier study (Romberg & Planert, 1970) indicated that children experienced much more difficulty in learning the subtraction algorithm than they did in learning the addition algorithm when they followed the traditional instructional sequence. This difficulty was expressed in either more instructional time required to obtain reasonable proficiency or failure to get group proficiency at all.

The usual addition and subtraction algorithms are related in several ways, both in terms of similarity of behaviors and in relationships in the mathematics involved. However, no dependency relationships are shown in the task analyses above certain common prerequisites (see Appendix A). Thus, the sets of behaviors associated with learning the two algorithms can be considered coordinate. The behaviors associated with learning the addition algorithm will be called X and those associated with learning the subtraction algorithm will be called Y. The usual order of teaching these algorithms is the instructional sequence X followed by Y, but the usual sequence is not the only reasonable sequence that could be constructed with respect to this hierarchy. In fact, as an alternative, an integrated sequence in which the algorithms would be introduced and developed together was suggested for the following reasons:

1) Instruction to this point in the experience of these children had emphasized the introduction and development of addition and subtraction together.

2) The regrouping associated with the addition algorithm as "carrying" is the reverse of the regrouping associated with subtraction as "borrowing."

3) Addition and subtraction of whole numbers are both counting processes.

4) Addition and subtraction as mathematics operations are inverses.

5) Expanded notation forms the basis of the development for both algorithms.

6) The common mechanical characteristics of "begin at the right" and vertical form suggest a parallel development of the algorithms.

Two sets of activities labeled S (for the standard addition then subtraction sequence) and I (for the integrated sequence) were accordingly developed, both with the common goal of providing instruction in the usual algorithms for addition and subtraction of two-digit whole numbers. The I activities can be characterized as a set of activities in which the algorithms are presented and developed concurrently. All daily activities place approximately equal emphasis on the operations of addition and subtraction. The mechanics and mathematics of "carrying" and "borrowing" are treated as a single entity, "regrouping."

The S activities were formed by separating each activity of I into an addition and subtraction component; each component was then completed to form a separate activity. The resulting activities were ordered in such a way that all of the addition activities were completed before the subtraction activities were begun. No mention of subtraction was to be found in the addition activities of S, but one or two addition items were included in each subtraction activity for the sole purpose of skill maintenance; however, no instruction in addition was intended during the course of the subtraction activities. The time of instruction, 20 days, was determined by the time required to complete the activities provided. (A sample of these materials appears in Appendix B.)

Since sequence I emphasizes relationships that sequence S does not even note, it could be argued that use of the integrated instructional sequence would result in higher achievement levels with the algorithms in general, and with the subtraction algorithm in particular, than would result from using S. Additionally, existing research evidence supports the use of integrated sequences in general. On the other hand, since the relationships between the sets of behaviors are not dependenc relationships, each algorithm can be learned without reference to the other. Furthermore, previous research with integrated sequences says nothing specifically about subjects in the seven- to nine-year age range nor about integrating tasks where the tasks are procedural chains. Integrating the two sets of behaviors might cause interference in the learning of two somewhat similar procedural chains that differ at crucial points.

Thus, while the evidence is scant, there is reason to examine whether or not an integrated instructional sequence for the algorithms might produce higher proficiency than the usual and somewhat unsatisfactory standard sequence.

Experimental Design

The experimental design used to compare the two sequences is diagrammed in Figure 1.

Group	Assignment	Treatment	Observation
I	R	(I)	$O_a + O_b$
S	R	(S)	$O_a + O_b$

Fig. 1. Experimental design to compare integrated (I) and sequential (S) instructional treatments on the addition and subtraction algorithms.

This design is a "true experimental design" (Campbell & Stanley, 1963) used to compare treatments.

Subjects

The subjects were the students of three second-grade classes at Randall Elementary School, Madison, Wisconsin, during the spring semester, 1970-71. Randall is a school in a well-established residential neighborhood. The parents of the children are largely professionally employed. All students had been participating in the tryout of materials for DMP. Two of these classes were brought together in terms of the prerequisites of the unit to be taught. The students were randomly reassigned to two groups to be taught by the two teachers who had been working with these children prior to reassignment. One of these groups was assigned to the sequential program (group S) and the other to the integrated program (group I). Group S contained 24 subjects and group I 25. The teachers were then randomly assigned to teach the two groups. These experimental groups began work on the same day at the same hour and maintained this relationship throughout the course of the study. This control of history was considered highly desirable since performance profiles were to be compared. Both groups I and S were judged by their teachers to have completed the activities by the 20th day of instruction.

A third class also experienced the integrated sequence. Data obtained from this group were used to check on the reactivity of the profile-generating tests, and to check on the general feasibility of using sequence I (Romberg & Wiles, 1972).

Observations

Both growth in ability to use the two algorithms and terminal performance were of interest. To examine growth (O_a), item sampling was employed (Lord & Novick, 1968). A 45-item pool consisting of 22 addition problems and 23 subtraction problems was partitioned via stratified random sampling into nine forms of five items each. (The forms are found in Appendix C.) The partitioning was subject to the constraints that each form contain at least two addition and two subtraction problems, exactly one verbal problem, at least one item requiring regrouping, and at least one item that does not require regrouping. The nine forms were randomly sequenced and those children who worked on these forms were randomly assigned a beginning point in the sequence in such a way that each form was used with nearly the same frequency.

The forms were administered by the teachers during the first part of the period beginning with the fifth instructional day and again every third instructional day. An instructional day was any day that arithmetic was taught. Each child was allowed three minutes to work on the form. His instructions were to do as well as he could but that he might not have time to complete the form. The group means that were estimated from these data were plotted across time, yielding an addition and subtraction profile for each group during instruction. All of the children in groups S and I worked on these test forms. Neither the children nor the teachers were informed of the correctness of any student responses prior to the end of the study. At that time summary reports of group performance were made available to the teachers.

To test terminal performance on the day following the 17th instructional day, all children in each group were administered a 20-item test (O_b) by their teachers which they were allowed 15 minutes to complete. Ten of these items are addition examples, five of which require "carrying," and ten are subtraction examples, five of which require "borrowing" in the usual algorithms. (See Appendix C for a copy of this test.) The 20 items were randomly ordered on the test form.

Conduct of the Study

During the course of instruction the teachers were instructed to keep a log of the activities being worked on each day along with any observations or personal judgments they felt should be noted. The investigator and other observers from the developmental staff of the project visited the classrooms at unannounced times (at least once a week) to verify that assigned programs were being followed. The personal judgments of all involved were that the experimental groups followed very closely both the prepared activities and the general intent of the two programs.

III
Results

The data are reported in two sections, the data from O_a followed by the data from O_b. The means reported for O_a are estimates based upon the scores of the individuals on the various forms. The significance tests reported are ruled significant if $p < .05$ and marginally significant if $.05 < p < .10$.

Data on Observations O_a

These are the scores from the periodic observations. All forms of the item-sampled test were used at each administration with each group in approximately equal numbers. If a child missed an administration, an estimated score was provided for him based upon his standing within the group at other administrations; eight such estimates out of a total of 294 scores were made. No more than two estimates of individuals' scores were necessary at any administration. Group addition, subtraction, and total scores at each administration are estimates formed by averaging the scores of the individuals of the group without regard for the test form used. The addition and subtraction scores are reported as proportions of unity. The total is the sum of these proportions. These data are summarized in Table 1.

The data for group I are summarized in Figure 2.

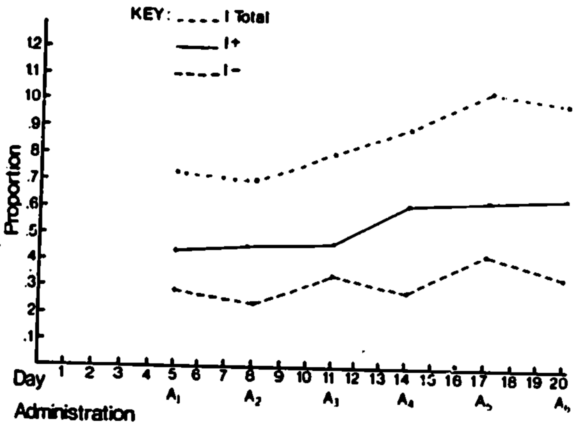

Fig. 2. Profiles of estimated means on O_a by administration for the integrated treatment group.

Table 1
Estimated Means and Total Means for Each Administration of O_a by Operation for the Integrated and Sequential Treatment Groups

	Treatment	A_1	A_2	A_3	A_4	A_5	A_6	Total
I	+	.44	.46	.47	.62	.63	.65	3.27
N = 25	-	.29	.25	.35	.29	.43	.36	1.96
	Total	.73	.71	.82	.91	1.05	1.01	5.23
S	+	.53	.75	.61	.69	.67	.74	3.99
N = 24	-	.33	.35	.33	.49	.47	.53	2.50
	Total	.85	1.10	.94	1.18	1.14	1.27	6.49

Addition performance is seen to be superior to subtraction performance at all administrations, and apart from a noticeable increase in performance on addition between the third and fourth administrations, the curves are similar in appearance. The addition performance ranges from .44 to .65 and increases steadily with administration. The subtraction performance varies from .25 to .43 and is not monotonically increasing.

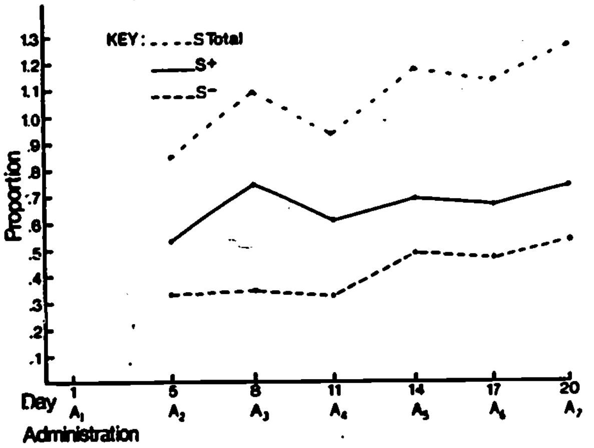

Fig. 3. Profiles of estimated means on O_a by administration for the sequential treatment group.

The data for group S are summarized in Figure 3. For this group, as for I, the addition performance is superior to subtraction performance at all administrations. The addition scores range from .53 to .75 while the subtraction scores range from .33 to .53. It is regrettable that the first administration occurred as late as the fifth day; however, a noticeable jump in performance with respect to addition can still be observed between the first and second administrations. Following this jump, addition performance is apparently stable near .75. The subtraction performance is stable at about .34 before the subtraction algorithm was studied; the performance then increases between the third and fourth administrations to a level of about .50. It should be noted at this point that the 45-item test that is the basis for O_a contains some items that do not require knowledge of the algorithms to be done; indeed, children who understand what is meant by addition, subtraction, and two-digit notation should be able to do many of these items if given enough time.

The effects of instruction are clearly discernible in the profiles of group S as their treatment began with instruction in the addition algorithm, with subtraction not mentioned until shortly after the third administration.

Figure 4 displays the addition profiles for the two groups. Differences apparently occur at the second administration; and though the differences are not always large, the scores for group S are superior to those for group I at all points.

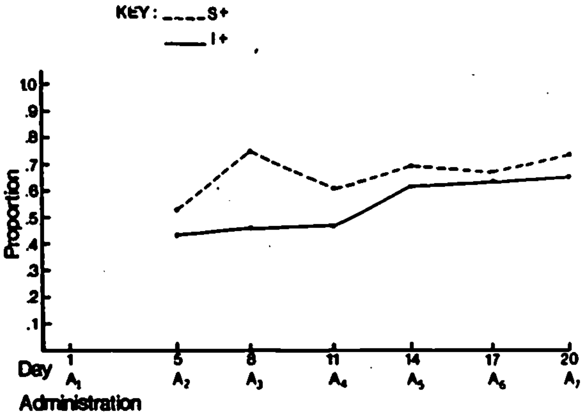

Fig. 4. Profiles of estimated means on O_a by administration for addition for the integrated and sequential treatment groups.

Figure 5 displays the subtraction profiles for both groups. Important differences appear to occur at the fourth and sixth administrations; and again, all comparisons but one favor group S.

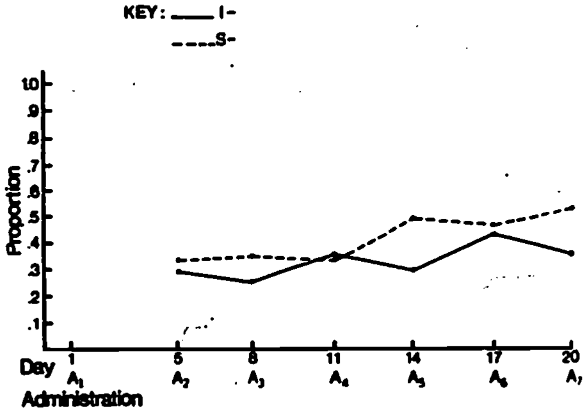

Fig. 5. Profiles of estimated means on O_a by administration for subtraction for the integrated and sequential treatment groups.

Table 2
Means on O_b by Operation and Regrouping
for the Integrated and Sequential Treatment Groups

	Addition			Subtraction			Total		
	No Carry	Carry	Total	No Borrow	Borrow	Total	No Regroup	Regroup	Total
$\underline{N}_I = 25$	4.12	2.60	6.72	3.24	.88	4.12	7.36	3.48	10.84
$\underline{N}_S = 23$	4.35	3.09	7.44	3.39	1.39	4.78	7.74	4.48	12.22

Table 3
Percent of Mastery (80% Criteria) on O_b by Operation and
Regrouping for the Integrated and Sequential Treatment Groups

	Addition			Subtraction			Total		
	No Carry	Carry	Total	No Borrow	Borrow	Total	No Regroup	Regroup	Total
$\underline{N}_I = 25$	.80	.36	.40	.56	.04	.08	.60	.04	.16
$\underline{N}_S = 23$	.82	.57	.57	.61	.13	.13	.65	.17	.22

Data on Observation O_b

The data for O_b are mean scores for each group based upon the 20-item test that was administered to all of the students on the 18th instructional day. The 20 items are partitioned into four sets: addition-no carry (+NOC), addition-carry (+C), subtraction-no borrow (-NOB), and subtraction-borrow (-B), where each partition contains five items. The group means are reported in Table 2 as they occurred in the range 0-5.

A mastery transformation was also applied to the raw scores of each individual: a score of 4 or 5 was changed to 1, and a score in the range 0-3 was changed to 0. The means of the resulting transformed scores are reported in Table 3. All comparisons favor group S.

Hypotheses Concerning O_a

The hypotheses associated with O_a concern differences among means, the existence of trends across administrations, and hypotheses concerning the relative difficulty of the operations.

Differences Among Group Means on O_a

The group means under consideration are the 12 estimated group means of addition and subtraction performance, two means for each of six administrations. The variables are labeled by administration and operation (e.g., 2+ is the estimated addition mean on the second administration). The basic analysis is provided by a multivariate analysis of variance in a repeated measures design and univariate analysis of variance for certain group comparisons of single scores. The hypotheses to be tested are:

H-1 The overall differences in group means on the 12 observations are zero.

H-2 The differences in total group means on the six administrations are zero.

H-3 The difference in the total group means summed across both operations and administrations is zero.

H-4 The difference in group means for addition summed across administrations is zero.

Table 4
Analysis of Variance of the Estimated Group Means on O_a
by Operation and Administration for Groups I and S

F-Ratio for Multivariate Test of Equality of Mean Vectors = .9849
$df = 12, 36$; $p < .4812$

Variable		Hypothesis Mean Square	Univariate F	p Less Than
1	1 +	.0943	.6357	.4293
2	2 +	1.0298	7.1833	.0102
3	3 +	.2325	1.4216	.2392
4	4 +	.0679	.5072	.4799
5	5 +	.0196	.1531	.6974
6	6 +	.0979	1.1314	.2930
7	1 -	.0193	.1617	.6895
8	2 -	.1245	.8968	.3485
9	3 -	.0050	.0471	.8292
10	4 -	.4871	3.5388	.0662
11	5 -	.0254	.2134	.6463
12	6 -	.3739	5.2141	.0270

Degrees of Freedom for Hypothesis = 1
Degrees of Freedom for Error = 47

H-5 The difference in group means for subtraction summed across administrations is zero.

Table 4 contains the multivariate analysis of variance associated with hypothesis 1 as well as univariate tests of the differences between each of the 12 pairs of means. The multivariate test does not allow rejection of the hypothesis of no overall group differences in these means; however, if only the univariate F's are considered, the differences between groups for the variables 2+ and 6- are significant ($p < .05$), and the differences for 4- are marginally significant ($p < .0662$). The MANOVA relating to H-2 is reported in Table 5. The multivariate F does not support the existence of overall differences across administrations, but in accordance with findings concerning H-1, the univariate differences are significant for the second administration ($p < .01$) and marginally so on the fourth ($p < .09$) and sixth ($p < .06$) administration.

Table 6 reports the ANOVA relating to H-3. The difference in the total performance summed across operation and administration is marginally significant ($p < .0562$) and favors group S.

Table 5
Analysis of Variance of the Estimated Group Means on O_a
Summed Across Operation by Administration for Groups I and S

F-Ratio for Multivariate Test of Equality of Mean Vectors = 1.4342
$df = 6, 42$; $p < .2246$

Variable	Hypothesis Mean Square	Univariate F	p Less Than
1 Sum 1	.1990	.6095	.4389
2 Sum 2	1.8704	6.5838	.0136
3 Sum 3	.1691	.4517	.5049
4 Sum 4	.9185	2.9434	.0929
5 Sum 5	.0896	.2508	.6189
6 Sum 6	.8545	3.6828	.0611

Degrees of Freedom for Hypothesis = 1
Degrees of Freedom for Error = 47

Table 6
Analysis of Variance of the Estimated Group Means on O_a Summed Across Operations and Administrations for Groups I and S

Variable	Mean Square	F	
Sum	19.4229	3.8356	$p < .0562$

Degrees of Freedom for Hypothesis = 1
Degrees of Freedom for Error = 47

The ANOVA's reported in Table 7 and Table 8 relate to H-4 and H-5, respectively. While no significant group difference is seen in the total subtraction performance, the difference in total addition performance is marginally significant ($p < .0540$).

In summary, no group differences can be seen in the multivariate space defined by either the 12 estimated group means or the 6 total estimated group means. However, the differences in addition performance at the second administration and in subtraction performance at the sixth administration are significant when only the univariate F is considered; similarly, the differences in subtraction at the fourth administration is marginally significant. The differences in total performance that favor group S are marginally significant and appear to be primarily due to differences in addition performance.

Hypotheses Concerning Trends Shown by O_a

The major hypothesis examined is:

H-6 There are no polynomial trends of degree less than six across administrations.

The variables used in this trend analysis are contrasts to estimate various components of trend over administrations. The variables ADLI, ADQD, ADCU, ADQT, and ADQN refer to the variance accounted for by a linear component, by a quadratic component after the linear component is removed, by a cubic component after the linear and quadratic components are removed, etc. The MANOVA reported in Table 9 tests the probability that the five components account for none of the total variance.

Table 7
Analysis of Variance of the Estimated Group Means for Addition on O_a Summed Across Administrations for Groups I and S

Variable	Mean Square	F	
Sum +	6.3380	3.9083	$p < .0540$

Degrees of Freedom for Hypothesis = 1
Degrees of Freedom for Error = 47

Table 8
Analysis of Variance of the Estimated Group Means for Subtraction on O_a Summed Across Administrations for Groups I and S

Variable	Mean Square	F	
Sum -	3.5706	2.3446	$p < .1325$

Degrees of Freedom for Hypothesis = 1
Degrees of Freedom for Error = 47

Table 9
Analysis of Variance from Zero of the Polynomial Components of Trend Across Administrations of O_a for the Total Estimated Means Summed Across Groups and Operations

F-Ratio for Multivariate Test of Equality of Mean Vectors = 4.0345
df = 5, 43; $p < .0044$

Variable	Hypothesis Mean Square	Univariate F	p Less Than
1 AD LI	4.2687	17.4365	.0002
2 AD QD	.0013	.0079	.9294
3 AD CU	.0172	.0942	.7603
4 AD QT	.0992	.4198	.5202
5 AD QN	.2052	.9690	.3300

Degrees of Freedom for Hypothesis = 1
Degrees of Freedom for Error = 47

The components are computed on the six means summing across groups and operations. The overall F requires that the null hypothesis be rejected ($p < .0044$). The univariate F's in Table 9 indicate that the linear component alone accounts for significantly more than zero variance, while the other components do not. A MANOVA was accordingly done on the component space determined by ADQD, ADCU, ADQT, and ADQN: this is reported in Table 10. The hypothesis that the total variance accounted for by these components is zero cannot be rejected.

Other tests were run to see if the trend components were different for groups or for operations with nonsignificant results.

In summary, there is a significant linear trend in the data that is not different for groups or for operations.

Hypotheses Concerning the Relative Difficulty of the Operations Shown by O.

The hypotheses to be tested are:

H-7 The relative difficulty of the two operations is the same.

H-8 The relative difficulty of the two operations is the same over treatment.

H-7 is related to Table 11. The variables OP/AD1, OP/AD2,...OP/AD6 are the differences between the estimated means for addition and subtraction summed across groups at each administration (e.g., OP/AD1 is the difference between the total estimated addition mean and the total estimated subtraction mean on the first administration). The overall F is signifi-

Table 10
Analysis of Variance from Zero of the Polynomial Components of Trend of Degree Greater than One Across Administrations of O_a for the Total Estimated Means Summed Across Groups and Operations

F-Ratio for Multivariate Test of Equality of Mean Vectors = .3107
df = 4, 44; $p < .8694$

Variable	Hypothesis Mean Square	Univariate F	p Less Than
1 AD QD	.0013	.0079	.9294
2 AD CU	.0172	.0942	.7603
3 AD QT	.0992	.4198	.5202
4 AD QN	.2052	.9690	.3300

Degrees of Freedom for Hypothesis = 1
Degrees of Freedom for Error = 47

Table 11
Analysis of Variance from Zero of the Differences Between the Means of the Operation Scores Summed Across Groups by Administration of O_a

F-Ratio for Multivariate Test of Equality of Mean Vectors = 15.5092
df = 6, 42; $p < .0001$

Variable	Hypothesis Mean Square	Univariate F	p Less Than
1 OP/AD1	.3832	7.3304	.0095
2 OP/AD2	1.0975	15.6643	.0003
3 OP/AD3	.5102	12.2659	.0011
4 OP/AD4	.9070	15.7202	.0003
5 OP/AD5	.4768	13.9480	.0006
6 OP/AD6	.7347	34.7736	.0001

Degrees of Freedom for Hypothesis = 1
Degrees of Freedom for Error = 47

cant ($p < .0001$) and the univariate F's at each administration are all significant beyond the .01 level. H-7 must be rejected.

The MANOVA of Table 12 and the ANOVA of Table 13 relate to H-8. Table 12 reports the MANOVA related to that of Table 11 where instead of summing across groups, the variables are differences of group differences. There is no evidence that these differences are significantly different from zero.

The variable + VS - of Table 13 is the result of subtracting the total addition and subtraction scores for each group, and then subtracting the differences. The null hypothesis is that + VS - is zero; H-8 cannot be rejected on the basis of the available evidence.

The Analysis of O_b

The hypotheses concerning the data from O_b are discussed in the following three categories: differences in group means, differences in group mastery, and differences in performance for the two operations.

Table 12
Analysis of Variance from Zero of the Differences of the Group Differences for Operations Means by Administration of O_a

F-Ratio for Multivariate Test of Equality of Mean Vectors = .7235
$df = 6, 42$; $p < .6331$

Variable	Hypothesis Mean Square	Univariate F	p < Less Than
1 OP/AD1	.0071	.1353	.7147
2 OP/AD2	.1096	1.5636	.2174
3 OP/AD3	.0765	1.8385	.1817
4 OP/AD4	.0478	.8289	.3673
5 OP/AD5	.0001	.0028	.9584
6 OP/AD6	.0223	1.0542	.3098

Degrees of Freedom for Hypothesis = 1
Degrees of Freedom for Error = 47

Table 13
Analysis of Variance from Zero of the Difference Between the Group Differences for Operations Summed Across Administrations of O_a

Variable	Mean Square	F	
+ VS -	.0986	.3218	$p < .5733$

Table 14
Analysis of Variance of the Group Means on O_b for Groups I and S

F-Ratio for Multivariate Test of Equality of Mean Vectors = .4793
$df = 4, 43$; $p < .7507$

Variable	Hypothesis Mean Square	Univariate F	p Less Than
1 + NO C	.6218	.5311	.4699
2 + C	2.8406	.9085	.3455
3 - NO B	.2742	.0876	.7687
4 - B	3.1317	1.6348	.2075

Degrees of Freedom for Hypothesis = 1
Degrees of Freedom for Error = 46

Differences in Group Means in O_b

The means under consideration are the four mean performances for each group on the variables of O_b. The hypotheses to be tested are:

H-1 There are no differences in group performance on the four variables.

Table 14 contains the MANOVA relating to H-1; there is no reason to reject this hypothesis even though all differences do favor group S.

Differences in Group Mastery Shown by O_b

The hypothesis of this section deals with the mastery data contained in Table 3. The hypothesis to be tested is:

H-2 There are no differences between the two groups in the proportions of those who demonstrate mastery on the four variables of O_b.

Table 15 contains the MANOVA relating to this hypothesis. The variables +NOCT2, +CT2, -NOBT2, and -BT2 are the proportions for addition-no carrying-mastery, addition-carrying-mastery, subtraction-no borrowing-mastery, and subtraction-borrowing-mastery, respectively. Even though all comparisons favor group S, the differences cannot be regarded as significant.

Differences in Performance for the Two Operations Shown by O_b

The hypotheses to be tested are:

H-3 There is no difference in performance for the two operations.

H-4 There is no difference in performance when regrouping is required or not required.

Table 15
Analysis of Variance of Mastery Scores on O_b by Group

F-Ratio for Multivariate Test of Equality of Mean Vectors = .6422
df = 4, 43; $p < .6354$

Variable	Hypothesis Mean Square	Univariate F	p Less Than
1 +NOCT2	.0082	.0513	.8218
2 + CT2	.5045	2.0335	.1607
3 -NOBT2	.0284	.1123	.7391
4 - BT2	.0980	1.2628	.2670

Degrees of Freedom for Hypothesis = 1
Degrees of Freedom for Error = 46

Table 16
Analysis of Variance of the Effects of Operation and Regrouping for the Data of O_b

F-Ratio for Multivariate Test of Equality of Mean Vectors = 74.6879
df = 3, 44; $p < .0001$

Variable	Hypothesis Mean Square	Univariate F	p Less Than
1 + VS -	82.6123	87.7548	.0001
2 RVSNOR	152.7105	105.2092	.0001
3 OP X R	7.4680	5.0187	.0300

Degrees of Freedom for Hypothesis = 1
Degrees of Freedom for Error = 46

H-5 There is no difference in mastery performance for the two operations.

H-6 There is no difference in mastery performance when regrouping is required or not required.

H-3 and H-4 are related to the MANOVA shown in Table 16. The variable + VS - is the difference between the mean for addition and the mean for subtraction, RVSNOR is the difference between the mean for regrouping and the mean for non-regrouping, and OP X R is the interaction between + VS - and RVSNOR; these variables are summed across both groups. The multivariate space determined by these variables is not zero ($p < .0001$), and both of the main effects are significant ($p < .0001$). Consequently both H-3 and H-4 are rejected. A rather curious finding here is that the interaction between the main effects of regrouping and operation is significant ($p < .03$); while this effect is small with respect to the main effects, it is not zero.

In accordance with the findings regarding H-1, Table 17 shows that the effects noted above are not different for the groups.

The MANOVA reported in Table 18 relates to H-5 and H-6. The variable OP T2 is the difference between the proportion of mastery demonstrated for the two operations, R T2 is the difference between the proportions of mastery demonstrated for regrouping and non-regrouping, and INT T2 is the interaction between the first two variables. Again the multivariate space is not zero ($p < .0001$), and both main effects are significant ($p < .0001$); consequently both H-5 and H-6 are rejected, but in this case the interaction is not significant ($p < .14$). The analysis reported in Table 19 indicates that these general effects are true for both groups; i.e., there are no group differences with respect to these three variables in accordance with the finding concerning H-2.

While the rejection of H-3, H-4, H-5, and H-6 is not surprising, it does establish that subtraction was more difficult than addition, and that regrouping was significantly more difficult than non-regrouping. Furthermore, these findings are not differentially affected by treatment.

Table 17
Analysis of Variance of the Differences in Group Means on O_b by Operation and Regrouping

F-Ratio for Multivariate Test of Equality of Mean Vectors = .2622
$df = 3, 44$; $p < .8523$

Variable	Hypothesis Mean Square	Univariate F	p Less Than
1 + VS -	.0082	.0087	.9263
2 RVSNOR	1.1480	.7909	.3785
3 OP X R	.0305	.0205	.8869

Degrees of Freedom for Hypothesis = 1
Degrees of Freedom for Error = 46

Table 18
Analysis of Variance of the Effects of Operation and Regrouping for the Mastery Transformation of O_b Data

F-Ratio for Multivariate Test of Equality of Mean Vectors = 39.8933
$df = 3, 44$; $p < .0001$

Variable	Hypothesis Mean Square	Univariate F	p Less Than
1 OP T2	4.4004	46.5940	.0001
2 R T2	8.6461	58.0724	.0001
3 INT T2	.2649	2.2404	.1413

Degrees of Freedom for Hypothesis = 1
Degrees of Freedom for Error = 46

The Interaction of Operation by Regrouping

A more interesting question at this point concerns why the interaction was significant in the untransformed data but not in the mastery data. The mastery transformation, T2, changed raw scores of 0 through 3 to 0, and raw scores of 4 or 5 to 1; the result was to eliminate the interaction of regrouping by operation. To look at the issue more closely, two other transformations, T3 and T4, were carried out and the interaction term examined.

T3 transforms scores 0 through 3 to 0, and leaves scores of 4 or 5 as they are; T4 leaves scores of 0-3 as they are and transforms 4 or 5 to 5. Tables 20 and 21 show the results of the analysis. The interaction is not significant for T3 ($p < .2009$) but is significant for T4 ($p < .0186$). While it may be argued that T4 and the raw data have larger sets of discrete values available (5 and 6, respectively) than do T2 and T3 (2 and 3, respectively), and so have a larger probability of an interaction, another possibility exists—namely that the interaction is primarily in the scores 0 through 3. If this is the case, it logically follows that students who have not mastered the algorithms find the regrouping associated with subtraction more difficult than that associated with addition, while those who have mastered the algorithms do not experience this differential difficulty. Such a finding would make planning instruction on the basis of data obtained from those who had already mastered the algorithms a questionable undertaking, and would underline the qualifications of the use of such data (e.g., Suppes, Jerman, & Dow, 1968). This question deserves further examination.

Table 19
Analysis of Variance of the Differences in Means for Group Mastery on O_b by Operation and Regrouping

F-Ratio for Multivariate Test of Equality of Mean Vectors = .4792
df = 3, 44; $p < .6984$

Variable	Hypothesis Mean Square	Univariate *F*	*p* Less Than
1 OP T2	.0254	.2694	.6063
2 R T2	.1461	.9813	.3271
3 INT T2	.0565	.4782	.4928

Degrees of Freedom for Hypothesis = 1
Degrees of Freedom for Error = 46

Table 20
Analysis of Variance of the Effects of Operation and Regrouping for the T3 Transformation of O_b Data

F-Ratio for Multivariate Test of Equality of Mean Vectors = 41.1544
df = 3, 44; $p < .0001$

Variable	Hypothesis Mean Square	Univariate *F*	*p* Less Than
1 OP T3	373.1044	43.7966	.0001
2 RGP T3	797.9800	70.6586	.0001
3 INT T3	15.3551	1.6839	.2009

Degrees of Freedom for Hypothesis = 1
Degrees of Freedom for Error = 46

Table 21

Analysis of Variance of the Effects of Operation and Regrouping for the T4 Transformation of O_b Data

F-Ratio for Multivariate Test of Equality of Mean Vectors = 65.5915

df = 3, 44; $p < .0001$

Variable	Hypothesis Mean Square	Univariate F	p Less Than
1 OP T4	393.6093	104.6244	.0001
2 RGP T4	669.3045	82.6275	.0001
3 INT T4	44.8023	5.9587	.0186

Degrees of Freedom for Hypothesis = 1

Degrees of Freedom for Error = 46

IV
Summary and Conclusions

All comparisons of group means based upon O_a and O_b but one (variable 3- of O_a) favor group S. Many of the tests reported in the hypotheses sections are, of course, not independent, and while several significant and many marginally significant differences have been found, the multivariate tests fail to show significant differences in overall group performance.

If the sum of all scores across the six administrations of O_a are taken as a measure of total learning during instruction, the differences favor group S and are primarily related to addition performance. This is not very surprising in view of the fact that group S was taught addition first and the children were asked to work one or two addition items for skill maintenance during the instructional activities dealing with subtraction. However, the differences for subtraction, even though nonsignificant ($p < .1325$), were also in favor of group S. This was not expected since group S was not exposed to instruction directly relating to the subtraction algorithm until shortly after the third administration of O_a, while group I used instructional activities dealing with both algorithms from the beginning.

Perhaps the most interesting results based upon O_a are that the effects of instruction are clearly discernible in the profiles, particularly for group S. It is tempting to interpret group S's first measure of subtraction performance, taken on the fifth instructional day, as a measure of transfer from experience with the addition algorithm; this is supported by a comparable level of performance of group I at this administration. This virtually indistinguishable performance for subtraction is maintained through the third administration, in spite of the fact that group I was receiving instruction with subtraction during the entire time. The absence of base-line data, however, makes such a claim tenuous. It may be that neither group showed improvement above initial performance through the third administration; further research is necessary to determine the issue.

The trend analysis based upon O_a reveals that the rate of learning is not only best described, but necessarily described, as linear. This is true for both groups and both operations. The lack of base-line data (not necessarily 0) and the lack of group mastery must be kept in mind when considering this finding; however, there is evidence that group learning was linear for the 20-day period of instruction of this study.

From the evidence provided by O_a it is concluded that if there are differences due to instructional sequence they favor group S and are primarily in addition performance. The data from O_a do not support the use of integrated sequences for children of these ages and with this type of material.

The evidence provided by O_b does not support inferences of group differences at the end of instruction even though all comparisons favor group S, but it does raise the interesting question of operation by regrouping interaction. The evidence from both O_a and O_b demonstrates that the addition algorithm is easier to learn than the subtraction algorithm, and that while regrouping is a major difficulty for both operations, it poses more of a problem for subtraction than it does for addition.

Limitations and Alternate Explanations

The findings of this study are limited by the lack of certain data and the general low level of mastery demonstrated by the students. Measuring performance immediately prior to instruction, transfer to related tasks following instruction, and retention of learned behaviors would have added greater interest to the post-test data and possibly would have allowed stronger inferences of group differences. More complete information would have been provided by profile data on the acquisition of all four

skills—addition without regrouping, addition with regrouping, subtraction without regrouping, and subtraction with regrouping—rather than just addition and subtraction. Such information might have allowed more precise determination of the source of possible group differences, and would have given a clearer indication of the importance of the operation by regrouping interaction found across groups.

Higher levels of group mastery are desirable, not only from the viewpoint of the adequacy of the instructional unit, but also for a more reliable test of the integrated instructional sequence. Important positive transfer from performance with the addition algorithm to performance with the subtraction algorithm would result in 50 to 80 percent of the children mastering the skills. Neither group reached this range in performance with the subtraction algorithm, while only group S reached this level with respect to the addition algorithm.

Some limitation on the interpretations of these data results from the possibility of effects due to teacher differences. The experimental design does not in itself control for such effects. The existence of any such effects is partially controlled by the activity approach to learning as used in this study, however. In this approach, the primary task of the teacher is to organize the specified activity. In the usual classroom usage of these activities, the teacher would be expected to modify, supplement, or replace certain activities to fit the particular needs of the class or personal style. Such modifications were not permitted in this study. Furthermore, observers were regularly in the classroom to insure that both the activities and particular interpretations of them were standardized across groups. In the judgment of these observers there was no reason to expect any effects due to differential teacher behavior. The only variable that was clearly manipulated was that of the sequence in which the activities were presented to the children. Both groups experienced the same activities, the same problems to solve, and were faced with the same discussion stimuli, only the sequence was different. However the hypothesis of teacher effect, while thus controlled for in some degree, cannot be categorically ruled out.

Interpretations of these data may be based upon considerations of hypothesized developmental levels of children. For example, it may be argued that children of these ages do not have the cognitive development necessary to profit from the similarities of the tasks to be learned, particularly when these similarities involve reverse relationships. It may be additionally argued that the tasks were too abstract for children of the developmental levels represented by these ages as evidenced by the generally low level of mastery attained.

The factor of age and developmental level is recognized as having some probable relationship to the question of an optimal instructional sequence for the learning of related procedural chains such as those represented by the addition and subtraction algorithms.

However, the specifications of any such relationship are not clear. The population of this study has been defined in terms of entering behaviors and age. In particular, a subject must have the prerequisite behaviors, but not the behaviors to be learned (as judged by his teacher in this study), and be 7 to 9 years of age. The relationship between these criteria and developmental level of the children is viewed as interesting and important, but is not the focus of this research.

Directions for Research

The question of the use of integrated sequences has been only partially answered. The sequence "subtraction then addition" was not tried. It is not known if instructional efficiency or total pupil knowledge is served by sequencing coordinate tasks on the basis of their relative difficulty. The relationships among age of subjects and type of task to sequencing strategy deserve further examination. The best sequencing strategy may be a function of age and developmental level as well as the type of task to be learned.

In general, neither of the instructional treatments produced acceptable levels of performance with the subtraction algorithm, and if the criterion is that 80% of the students master 80% of the material, the instruction with the addition algorithm is not acceptable either. It is not known how the learning profiles would be affected if instruction were continued to higher mastery levels.

There is evidence in this study that there may be important differences in the way one who has mastered a task and one who has not view a task; further investigation into this question also seems desirable.

Further research is being undertaken that incorporates a wider range of dependent variables as well as measures of initial performance. This research involves the use of revised and expanded materials that should result in generally higher achievement levels.

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Appendix A
Behaviors Associated with Tasks Used in This Study

<u>Terminal Behavior</u> AS - T (Addition-Subtraction)
Given a set of examples involving both addition and subtraction of two two-digit whole numbers, computes the required sums of differences using the appropriate algorithm. The sums are to be restricted to those less than 100.

<u>Task A</u> (Addition)

1. Given the numeral phrase $qb + sb$, states the sentence $qb + sb = (q + s)b$.

2. Given the numerals $qb + r$ and $sb + t$ in expanded notation, states the sentence

$$\begin{array}{cccc} q & b & + & r \\ s & b & + & t \\ \hline (q+s) & b & + & (r+t) \end{array}$$

3. Given the numeral $qb + r$, where $r \geq b$, states the sentence $qb + r = (q + 1)b + (r - b)$.

4. Given the numerals $qb + r$ and $sb + t$ in expanded notation and having written the sentence $(qb + r) + (sb + t) = (q + s)b + (r + t)$, finds numerals x and y such that $(q + s)b + (r + t) = xb + y$ where $y < b$, and writes the sentence

$$\begin{array}{c} qb + r \\ sb + t \\ \hline xb + y \end{array}$$

5. Given two two-digit numerals a and b written in compact notation, writes these numerals in expanded notation, finds their sum and the numeral c in compact notation which represents this sum, and writes the sentence $a + b = c$.

6. Given two two-digit numerals a and b written in compact notation, applies the addition algorithm to a and b to find the numeral c in compact notation which represents this sum and writes the sentence $a + b = c$.

<u>Task S</u> (Subtraction)

1. Given the numeral phrase $qb - sb$, where $q \geq s$, states the sentence $qb - sb = (q - s)b$.

2. Given the numeral $qb + r$, states the sentence $qb + r = (q - 1)b + (b + r)$.

3. Given two numerals $qb + r$ and $sb + t$, where $qb + r \geq sb + t$, compares r and t to determine whether $r \geq t$ or $r < t$.

4. Given two numerals $qb + r$ and $sb + t$, where $qb + r > sb + t$, and having determined that $r < t$, rewrites $qb + r$ as $(q - 1)b + (r + b)$, finds the differences $(q - 1) - s$ and $(r + b) - t$, and writes the sentence

$$\begin{array}{cccc} q & b & + & r \\ -s & b & - & t \\ \hline (q-1-s) & b & + & (r+b-t) \end{array}$$

5. Given two numerals $qb + r$ and $sb + t$, where $qb + r \geq sb + t$, and having determined that $r \geq t$, finds the differences $q - s$ and $r - t$ and writes the sentence

$$\begin{array}{cccc} q & b & + & r \\ -s & b & - & t \\ \hline (q-s) & b & + & (r-t) \end{array}$$

6. Given two two-digit numerals a and b

written in compact notation, where $a \geq b$, writes these numerals in expanded notation, finds the difference $a - b$ and the numeral c in compact notation which represents this difference, and writes the sentence $a - b = c$.

7. Given two two-digit numerals a and b written in compact notation, where $a \geq b$, applies the subtraction algorithm to the difference $a - b$ to find the numeral c in compact notation which represents this difference and writes the sentence $a - b = c$.

Entering Behaviors

1. Given two two-digit numerals a and b written in compact notation, states that $a > b$ (or $b < a$).

2. Given a numeral between 0 and 99 in compact notation, symbolically models this numeral in expanded notation.

3. Given two numbers a and b whose sum is less than or equal to 20 (base 10), having represented the sum of a and b and having found a numeral c in compact notation which is equal to the sum of a and b, writes the sentence $a + b = c$.

4. Given two numbers a and b such that $0 \leq a < 20$, $0 \leq b < 10$, and $b \leq a$, having represented the difference $a - b$ and having found a numeral c in compact notation which is equal to that difference, writes the sentence $a - b = c$.

5. Given two numbers a and b whose sum is less than or equal to 20 (base 10), states the compact number name for $a + b$ and the complete sentence $a + b = c$.

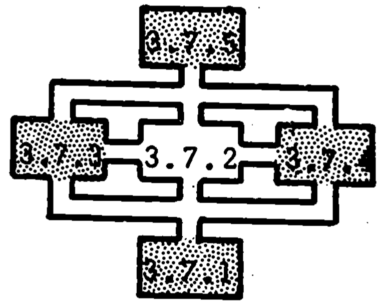
6. Given two numbers a and b such that $0 \leq a < 20$, $0 \leq b < 10$, and $b \leq a$, states the compact number name for $a - b$ and the complete sentence $a - b = c$.

Appendix B
A Sample of the Instructional Activities Used in This Study

1, 2, 3, 4, 5

Activity 3.7.2

Individual/Small Group



●● MATERIALS NEEDED: Worksheet 3.7.2
Objects to be used as counters
Number arrays
A "store"

PREPARATION:

3.7.2

1) Set up a play store, all of whose items are priced below 50¢. This can be done in several ways. You can use items in the room, allowing the children to do the selecting and pricing, e.g., an eraser for 33¢. You can have the children bring empty containers from home and reprice them if necessary. You can have several stores—a bakery, a grocery, a toy store, a candy store, a place to eat, etc., each with four or five items for sale. The children can draw pictures of or make the items to be sold.
2) Duplicate sufficient copies of the worksheet.

DESCRIPTION

There are several ways in which this activity can be done, depending somewhat on the type of store you and the children set up.

If you use several stores, assign one child as a storekeeper for each store and let the rest of the class be shoppers. The shoppers should select two items from a store. The storekeeper will fill in the cost of the two items on the worksheet and return the items to the store. Then the shopper must calculate the total cost. Next, the shopper finds out how much money he would have left after paying for the items if he started with the amount shown on his worksheet. If he doesn't have enough, he is to find out how

much more he would need. This will require some explanation by you. In the last column, the child should cross out the words that do not apply. The shoppers move from store to store, with the children switching roles after all have had a reasonable amount of time to reach several stores.

If you use just one store, at least two possibilities exist. To avoid the confusion of all the children going to the store at once, you might let it be self-service, with the children going to the store at any convenient time during the day. The child is to select two items, write down the price of each, and then figure out how much money he must pay. He then finds the amount of money that he would have left or how much more he needs. Finally he returns the items to the store. The student is to repeat the process (over a period of time) until he has completed the activity sheet.

A second possibility would be to have the pupils work in pairs or threesomes, with one being a storekeeper or clerk for the other two. The storekeeper would find the items and calculate the total cost, with the shopper then determining his financial status. Each child in the group should get an opportunity to be the storekeeper.

If you use just one store, it is suggested that you place it in a corner of the room. While some of the children are using the store, the others can be engaged in some of the activities described in Activity 3.7.3 and 3.7.4. The children can then alternate among the three activities until each child has had an opportunity to work on all three.

As in Activity 3.7.1, counting devices should be available and grouping by tens encouraged.

work sheet 3.7.2

name ____________________

You have	How much must you pay?	Do you have enough?	
55¢		Yes No	I have _____¢ left. I need _____¢ more.
89¢		Yes No	I have _____¢ left. I need _____¢ more.
65¢		Yes No	I have _____¢ left. I need _____¢ more.
35¢		Yes No	I have _____¢ left. I need _____¢ more.
49¢		Yes No	I have _____¢ left. I need _____¢ more.
75¢		Yes No	I have _____¢ left. I need _____¢ more.

GPO 829-380-5

Activity 3.7.2

1, 3, 5

Individual/Small Group

Materials Needed

Worksheet 3.7.2
Objects to be used
as counters
Number arrays
A "store"

Preparation

1. Set up a play store or stores, all of whose items are priced below 50¢. This can be done in several ways. You can use items in the room, allowing the children to do the selecting and pricing, e.g., an eraser for 33¢. You can have the children bring empty containers from home and reprice them if necessary. You can have several stores -- a bakery, a grocery, a toy store, a candy store, a place to eat, etc., each with four or five items for sale. The children can draw pictures of or make the play items to be sold.

2. Duplicate sufficient copies of the worksheet.

3. Save these materials for use with subtraction later.

Description

There are several ways in which this activity can be done, dependent somewhat on the type of store you and the children set up.

If you use several stores, assign one child as a storekeeper for each store and let the rest of the class be shoppers. The shoppers should select two items from a store. The storekeeper will fill in the cost of the two items on the worksheet and return the items to the store. Then the shopper must calculate the total cost. This will require some explanation by you. The shoppers move from store to store, with the children switching roles after all have had a reasonable amount of time to reach several stores.

If you use just one store, at least two possibilities exist. Due to the confusion possible if all the children go to the store at once, you might let it be self-service, with the children going to the store at any convenient time

during the day. The child is to select two items, write down the price of each, and then figure out how much money he must pay. Finally, he returns the items to the store. The student is to repeat the process (over a period of time) until he has completed the activity sheet.

A second possibility would be to have the pupils work in pairs or threesomes, with one being a storekeeper or clerk for the other two. The storekeeper would find the items and calculate the total cost. Each child in the group should get an equal opportunity to be the storekeeper.

If you use just one store, it is suggested that you place it in one corner of the room. While some of the children are using the store, the others can be engaged in some of the activities described in Activity 3.7.3 and 3.7.4. The children can then alternate among the three activities until each child has had an opportunity to work on all three.

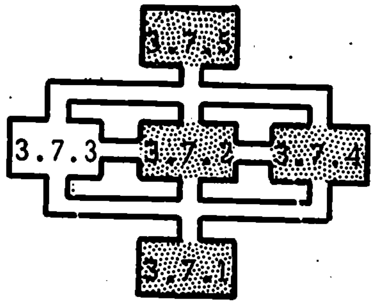
As in Activity 3.7.1, counting devices should be available and grouping by tens encouraged.

1, 2, 3, 4, 5

Activity 3.7.3

Individual/Pair

MATERIALS NEEDED: Activity Cards 3.7.3 a to p
Worksheet 3.7.3 a, b
Cloth bags
Unifix cubes
Discs
Beans, corn
Lead washers
Lots-a-Links
Buttons
Number arrays (Worksheet 3.6.7)



PREPARATION

3.7.3

1) Duplicate worksheets in needed quantities.

2) Set up 16 work stations around the room, each identified with a letter, a through p. At each station place two bags labeled One and Two filled with objects as shown on the list. Wherever possible, as with Lots-a-Links or cubes, they should be grouped by tens with the "left-overs" remaining unattached. The children should help fill the bags, with the first group at each station doing the work. Also, place the corresponding card at each station.

Station	Objects	Amount in Bag One	Amount in Bag Two
A	Washers	23	44
B	Beans	25	32
C	Lots-a-Links	45	18
D	Discs	7	22
E	Unifix cubes	14	17
F	Corn	35	21
G	Buttons	6	23
H	Washers	19	8
I	Lots-a-Links	63	25

Station	Objects	Amount in Bag One	Amount in Bag Two
J	Discs	43	17
K	Unifix cubes	31	46
L	Discs	30	20
M	Lots-a-Links	26	17
N	Beans	41	15
O	Buttons	30	20
P	Washers	24	25

Note: Substitute materials may be used where necessary.

DESCRIPTION

The children are to move around to each station, perform the task described on the corresponding card, and return the materials to their original state. Responses are written in the appropriate space on the worksheet. As you circulate among the children, encourage counting by tens. You should note whether or not they are using the correct process (addition or subtraction) for the task described. You should also have them occasionally verify their work. It is not necessary that they do all of the cards, but they should do at least half.

Differences in ability to find correct sums and differences should start becoming apparent. Be alert for this. This may be the time to start giving formal instruction in the use of the algorithms to some of your students. If so, suggestions are given in Activity 3.7.5. You may also wish to group children into pairs or threesomes in which one of the more advanced pupils could act as a helper or peer-tutor for the others.

name

work sheet 3.7.3.

a		e	
b		f	
c		g	
d		h	

work sheet 3.7.3b

name

i		m	
i		n	
k		o	
l		p	

Activity 3.7.3.S

2, 4, 5 Individual/Pair

Materials Needed

Activity cards 3.7.3
Worksheet 3.7.3.S
Cloth bags
Metal tags
Unifix cubes
Beans, corn
Lead washers
Lots-a-Links
Buttons
Number arrays

Preparation

1. Set up 8 work stations around the room, identified with the letters A-H. At each station place two bags labeled 1 and 2 filled with objects as shown on the list. Wherever possible, as with Lots-a-Links or cubes, they should be grouped by tens with the "left-overs" remaining unattached. The children could help to fill the bags, with the first group at each station doing the work. Also, place the corresponding card at each station.

Station	Objects	Amount in Bag One	Amount in Bag Two
A	Beans	25	32
B	Unifix cubes	14	17
C	Corn	35	21
D	Lots-a-Links	63	25
E	Unifix cubes	31	46
F	Beans	41	15
G	Buttons	30	20
H	Washers	24	25

Note: Substitute materials may be used where necessary.

2. Duplicate sufficient copies of Worksheet 3.7.3.S.

Description

The children are to move around to each station, perform the task described on the corresponding card, and return the materials to their original state. As you circulate among them, encourage counting by tens. You should also note

whether or not they are using the correct process (subtraction) for the task described. You should also have them occasionally verify their work. It is not necessary that they do all of the cards but they should do at least half.

As with the previous addition activity, differences in ability should start becoming apparent. Be alert for this. This may be the time to start giving formal instruction in the use of the algorithms to some of your students. You may also wish to group children into pairs or triples in which one of the more advanced pupils could act as a helper or peer-tutor for the others.

GPO 829-380-4

name

work sheet 3,7,3,S

a		e	
b		f	
c		g	
d		h	

Appendix C
The Tests Used in This Study

Teacher ______________________ Name ______________________

75 +24	79 -31	50 +40	72 -15	71 +26
40 -32	89 -46	29 +38	41 -22	39 +49
43 -30	56 +16	73 -52	79 +18	3 +16
97 -77	50 -36	5 +85	93 -26	22 +67

GPO 829-380-3

3.7.1

Write the missing number.

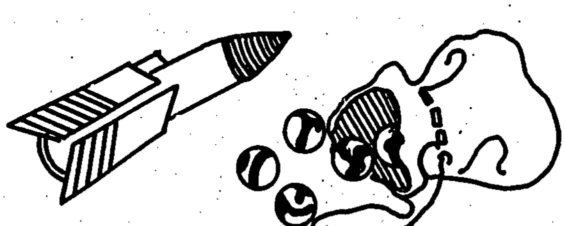
Name

57
-4

3
+9

39
+41

30
-19

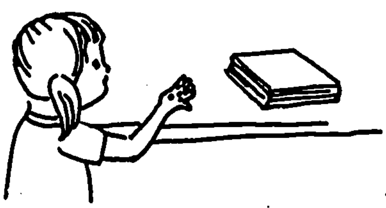


Bob had 48 marbles. He traded 21 of them for a toy rocket. How many marbles does he still have?

3.7.2

Write the missing numbers.

Name

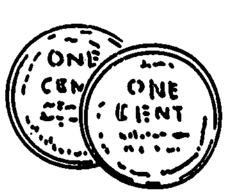
$89 - 7 = 80 + \square$	$\begin{array}{r} 85 \\ +4 \\ \hline \end{array}$
$\begin{array}{r} 38 \\ +16 \\ \hline \end{array}$	$\begin{array}{r} 64 \\ -56 \\ \hline \end{array}$
 Jane has 42 pennies. How many more pennies must she save before she can buy a book that costs 75¢?	

3.7.3

Write the missing numbers.

Name

$\begin{array}{r} 78 \\ -43 \\ \hline \end{array}$	$\begin{array}{r} 32 \\ +67 \\ \hline \end{array}$
$\begin{array}{r} 92 \\ -14 \\ \hline \end{array}$	$\begin{array}{r} 15 \\ +66 \\ \hline \end{array}$



Mother had some pennies. After she gave 9 of them to Cindy, she had 23 pennies left. How many pennies did she have before she gave some to Cindy?

3.7.4

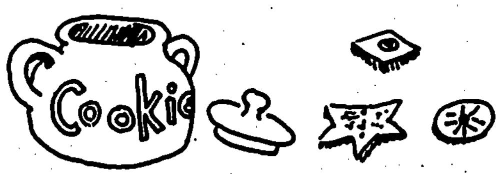
Write the missing numbers. Name

$63 + 5 = \boxed{} + 8$

$$\begin{array}{r} 17 \\ -\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} 84 \\ -46 \\ \hline \end{array}$$

$$\begin{array}{r} 41 \\ +29 \\ \hline \end{array}$$



Mother put 4 round cookies, 7 square cookies, and 5 star cookies in the cookie jar. How many cookies in all did she put in the cookie jar?

3.7.5

Write the missing numbers.

____
Name

$12 + 73 = 80 + \boxed{}$

$$\begin{array}{r} 36 \\ -\ 7 \\ \hline \end{array}$$

$$\begin{array}{r} 37 \\ +44 \\ \hline \end{array}$$

$$\begin{array}{r} 47 \\ -19 \\ \hline \end{array}$$

Jack found 37 rocks. He found 15 more than Jim found. How many rocks did Jim find?

3.7.6

Write the missing numbers.

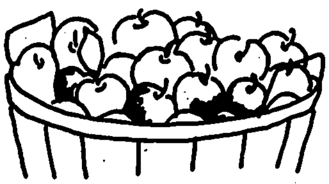
Name

$$\begin{array}{r} 96 \\ -52 \\ \hline \end{array}$$

$97 - 54 = \square + 3$

$$\begin{array}{r} 55 \\ +28 \\ \hline \end{array}$$

$$\begin{array}{r} 63 \\ +32 \\ \hline \end{array}$$



After Nancy took 10 apples from a basket, there were 30 apples left. How many apples were in the basket before Nancy took some?

3.7.7

Write the missing numbers.

Name

$$\begin{array}{r} 35 \\ 12 \\ +41 \\ \hline \end{array}$$

$9 + 6 = 10 + \boxed{}$

$$\begin{array}{r} 83 \\ -64 \\ \hline \end{array}$$

$$\begin{array}{r} 96 \\ -72 \\ \hline \end{array}$$

Yesterday father drove his car 26 miles. Today he drove 42 miles. How many miles did he drive in these two days?

3.7.8

Write the missing numbers.

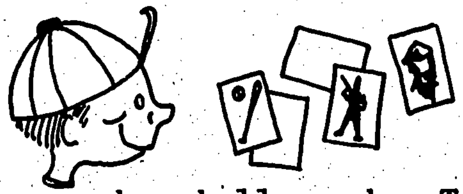
Name

$$\begin{array}{r} 18 \\ +\ 5 \\ \hline \end{array}$$

$13 - 8 = 10 - \boxed{}$

$$\begin{array}{r} 36 \\ +\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 82 \\ -37 \\ \hline \end{array}$$



Bill saves baseball cards. Today he got 15 new cards. Now he has 75 cards. How many baseball cards did Bill have yesterday?